Multimaterial topology optimization of a heated channel

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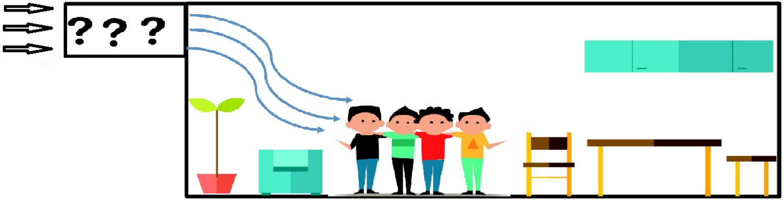
9th July 2021

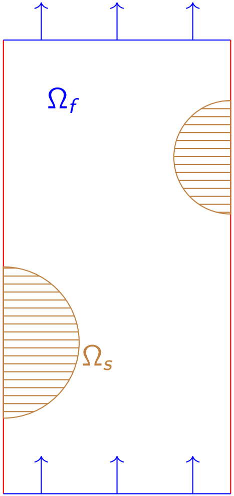
Introduction



How to design the device to optimize the physical phenomenon?

- minimize temperature at output
- minimize pressure drop
- ...

Model

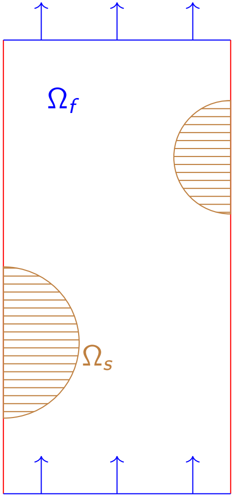


In a region Ω, suppose a flow occupies a region Ω_f and the solid defines a region Ω_s such that $\Omega = \Omega_f \cup \Omega_s$. Suppose also that the channel is heated.

$\Longrightarrow$ Navier-Stokes coupled with advection-diffusion equations.

(Focus only on steady-state model.)

Model

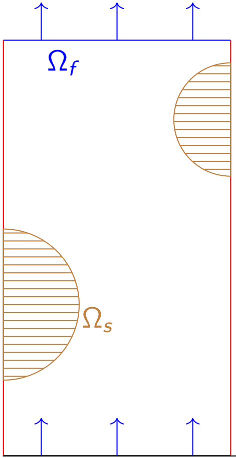


$$
\begin{aligned}
\nabla \cdot \mathrm{u} &= 0 \quad \text{in } \Omega_f \\
(\mathrm{u} \cdot \nabla)\mathrm{u} + \nabla p - \mathrm{Re}^{-1}\Delta \mathrm{u} - \mathrm{Ri}\theta e_y &= 0 \quad \text{in } \Omega_f \\
\nabla \cdot (\mathrm{u}\theta) - \nabla \cdot (\mathrm{Re}^{-1}\mathrm{Pr}^{-1}k(x)\nabla\theta) &= 0 \quad \text{in } \Omega \\
\mathrm{u} &= 0 \quad \text{in } \Omega_s
\end{aligned}
$$

We would like to control the distribution of solid. However, hard to put this in a control context.

$\implies$ Penalization technique.

Model



$$\nabla \cdot \mathrm{u} = 0 \quad \text{in } \Omega$$

$$(\mathrm{u} \cdot \nabla)\mathrm{u} + \nabla p - \mathrm{Re}^{-1}\Delta \mathrm{u} - \mathrm{Ri}\theta e_y + \eta 1_{\Omega_s} u = 0 \quad \text{in } \Omega$$

$$\nabla \cdot (\mathrm{u}\theta) - \nabla \cdot (\mathrm{Re}^{-1}\mathrm{Pr}^{-1} k(x, 1_{\Omega_s})\nabla\theta) = 0 \quad \text{in } \Omega$$

Converges as $\eta \to +\infty$ [1].
However, the indicator 1_{Ω_s} is a binary function, so not suitable for optimization.
$\implies$ Convexification and penalization of intermediate values.

[0][1] Angot, P., Bruneau, C.H., Fabrie, P.: A penalization method to take into account obstacles in incompressible viscous flows. Numerische Mathematik, 1999

How to penalize intermediate values ?

Convexification. $\{0,1\} \to [0,1]$.

↓

Define $\alpha : \Omega \to [0,1]$ and $h_\tau : [0,1] \to [0,1]$, s.t. $h_\tau(\alpha(x)) \approx 1_{\Omega_s}(x)$

↓

How to avoid the values between 0 and 1?

↙ ↘

Optimization approach: add a cost to push towards 0 or 1.

Dynamical approach: use a function h_τ that will quickly evolve towards 0 or 1.

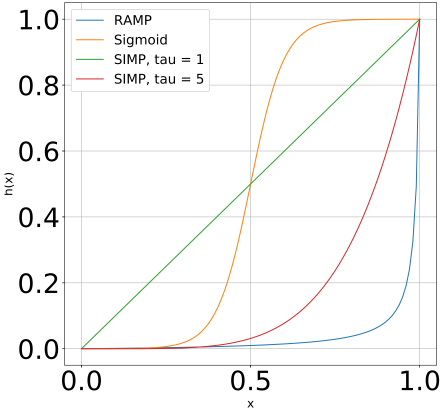
How to penalize intermediate values ?

Cost penalization : usual concerning optimization with constraints (barrier methods...). In our case, we minimize the following cost:

$$\int_{\Omega} h_{\tau}(\alpha(x))(1 - h_{\tau}(\alpha(x)))dx$$

Since $h_{\tau} : [0, 1] \rightarrow [0, 1] \implies$ penalizes the values between 0 and 1, and $h_{\tau}(\alpha)$ approximates 1_{Ω_s}.

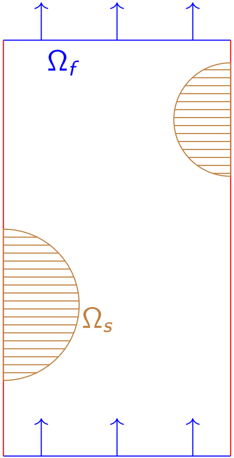
How to penalize intermediate values ?



h_τ can be used to penalize intermediate values, and obtain $h_\tau \xrightarrow[\tau\to+\infty]{} 0$ or 1.

- SIMP : $h_\tau(x) = x^\tau$
- RAMP : $h_\tau(x) = 1 - (1-x)^{\frac{1+\tau}{1+\tau-\tau x}}$
- Sigmoid : $h_\tau(x) = \frac{1}{1+\exp(-\tau(x-x_0))} - \frac{1}{1+\exp(\tau x_0)}$

Model



$$\nabla \cdot \mathrm{u} = 0 \quad \text{in } \Omega$$

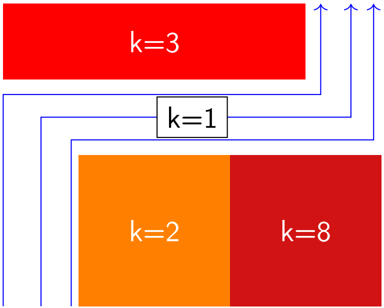
$$(\mathrm{u} \cdot \nabla)\mathrm{u} + \nabla p - \mathrm{Re}^{-1}\Delta \mathrm{u} - \mathrm{Ri}\theta e_y + \eta h_\tau(\alpha) u = 0 \quad \text{in } \Omega$$

$$\nabla \cdot (\mathrm{u}\theta) - \nabla \cdot (\mathrm{Re}^{-1}\mathrm{Pr}^{-1} k(x, 1_{\Omega_s})\nabla\theta) = 0 \quad \text{in } \Omega$$

$\Longrightarrow$ α defines the distribution of solid.

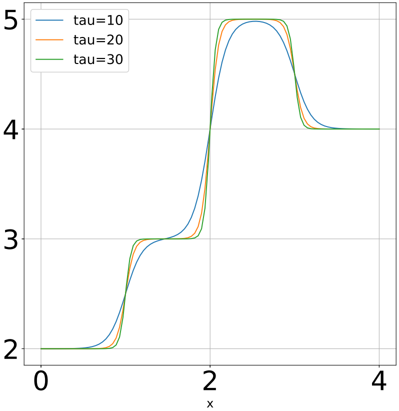
Another problem: how does this affect the conductivity function $k(x, 1_{\Omega_s})$?

Model



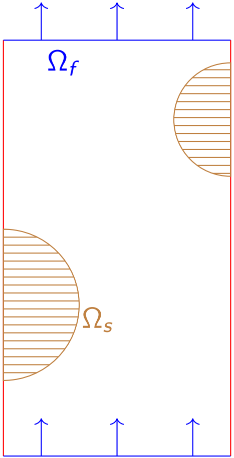
Usually, thermal conductivity = discrete set of constants.
We would like to control the distribution of thermal conductivity also!

Interpolation of thermal conductivity constants



$\Longrightarrow$ Advocates for the use of the sigmoid interpolation function. Introduces a new control function ϕ.

Model



We would like to solve:

$$\min_{\alpha,\phi} \mathcal{J}(u, \theta, p)$$

such that:

$$\begin{aligned} \nabla \cdot \mathrm{u} &= 0 \quad \text{in} \ \ \Omega \\ (\mathrm{u} \cdot \nabla)\mathrm{u} + \nabla p - \mathrm{Re}^{-1}\Delta \mathrm{u} - \mathrm{Ri}\theta e_y + \eta h_\tau(\alpha) u &= 0 \quad \text{in} \ \ \Omega \\ \nabla \cdot (\mathrm{u}\theta) - \nabla \cdot (\mathrm{Re}^{-1}\mathrm{Pr}^{-1} k_\tau(\alpha, \phi)\nabla\theta) &= 0 \quad \text{in} \ \ \Omega \end{aligned}$$

α defines the distribution of solid, ϕ the thermal conductivity.

Existence of solution

We first study the existence of solutions to these equations. Suppose :

$$\alpha, \phi \in \mathcal{U}_{\text{ad}} = \{\xi \in BV(\Omega) \mid 0 \leq \xi(x) \leq 1 \mid |D\xi|(\Omega) \leq \kappa\}.$$

and h_τ, k_τ are bounded continuous functions with $k(x) \geq k_{\text{min}} > 0$.

Theorem

Given α, $\phi \in \mathcal{U}_{\text{ad}}$, and given Ri, input velocity and heat flux small enough, there exists a solution $(\mathrm{u}, \theta, p) \in H^1(\Omega)^2 \times H^1(\Omega) \times L^2(\Omega)$.

Optimality problem

Suppose now we would like to minimize a functional $\mathcal{J}(\alpha, \phi, u, \theta, p)$ Under some hypothesis, there exists an optimal solution. But more interesting:

Theorem

Define a sequence $(\alpha_h^*, \phi_h^*, u_h^*, \theta_h^*, p_h^*)$ of global optimal solutions to the discretized problem. Then it converges (weak-*, weak-*, strong, strong, strong) to a global optimal solution of the continuous problem.

Numerical example

$$\max_{\alpha,\phi} \int_{\Gamma_{\text{out}}} \theta$$

$$\text{s.t.} \begin{cases} (u, \theta, p) \text{ solution of N.-S. + adv-diff} \\ u_{\text{in}}(x,0) = 1.8x(1-x), \varphi = 3 \\ Re = 100, Ri = 2, Pr = 0.71, \eta = 10^8. \end{cases}$$

Code done using Fenics, available online.

Numerical example

You may have noticed that the control ϕ depends on the control α, since the distribution of the thermal diffusivity in the solid depends.... On the distribution of solid.

$\implies$ Advocates for alternating directions!

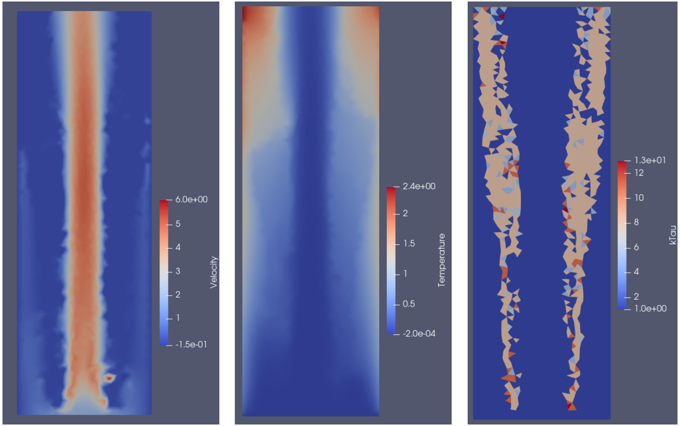
Optimize first on α, then on ϕ. In all numerical tests: it works way better.

Numerical example

For this presentation, I want to compare two approaches:

1. Gradually increase τ defining h_τ and k_τ.
2. Keep τ fixed and penalize more and more in the cost $\int_\Omega h_\tau(\alpha)(1 - h_\tau(\alpha))$.

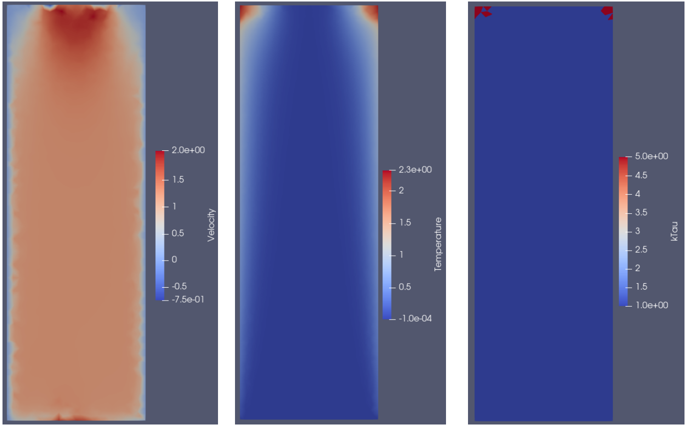
Numerical example: function



(a) Velocity (b) Temperature (c) k_τ

Figure: Result with function penalization ($\tau \to +\infty$).

Numerical example: cost



(a) Velocity (b) Temperature (c) k_τ

Figure: Result with cost penalization.

Conclusion

How to solve a complex topology optimization problem applied to fluid dynamics?

- A nice and coherent way to approximate the problem.
- Mathematically sound, and now fully analyzed.
- Gives interesting numerical results.
- Gives hints concerning ways to enhance it!

Conclusion

What is left to enhance?

- Computations really long, may hardly converge.
- Works for some examples, and not for others: does their exist a uniform method for all problems?
- Optimal solution for the thermal conductivity still depends on the initial guess: how to enhance this?
- Solution needs to be "cleaned" (how do we remove the holes without altering the optimality?).

We have a preprint!

Topology Optimization for Steady-state anisothermal flow targeting solid with piecewise constant thermal diffusivity
Alexandre Vieira, Alain Bastide, Pierre-Henri Cocquet
`https://hal.archives-ouvertes.fr/hal-02569142`

Question time!